

ECEn 555, Fall 2009

Homework #1

Due September 14, 5:00 pm

- 1.1 Calculate the maximum core diameter needed for a circular dielectric waveguide (refractive index 1.5) to support a single mode of radiation with wavelength 1 μm when (a) the waveguide has no cladding and (b) the waveguide is of the step index type with $\Delta = 0.01$. Comment on the values obtained from the point of view of manufacturing single mode waveguides.
- 1.2 A laser operating at 1.5 μm with a linewidth of 2 nm is used in conjunction with a single mode silica-based fiber. Estimate the material dispersion expected.
- 1.3 Show that when the source linewidth is exactly centered on the wavelength ($=\lambda_m$), where $\lambda_o^2(d^2n/d\lambda_o^2) = 0$, then the resulting material dispersion can be written

$$\Delta\tau = \frac{L}{c} \left[\lambda_m^3 \left(\frac{d^3n}{d\lambda^3} \right)_{\lambda=\lambda_m} \right] \left(\frac{\Delta\lambda}{\lambda_m} \right)$$

(Hint: the simplest way of proceeding is via the group index $N_g = n - \lambda_o \frac{dn}{d\lambda_o}$; at λ_m , N_g

will have a minimum value and hence about λ_m will have a $(\lambda_o - \lambda_m)^2$ dependence on λ_o . Thus we may write $N_g(\lambda_o) - N_g(\lambda_m) = [(\lambda_o - \lambda_m)^2/2!](d^2n/d\lambda^2)_{\lambda_m}$.)

1.4 **Refractive index and the Sellmeier dispersion equation.** The Sellmeier dispersion equation is an empirical expression for the refractive index of glass in terms of the wavelength λ ,

$$n^2 - 1 = G_1 \lambda^2 / (\lambda^2 - \lambda_1^2) + G_2 \lambda^2 / (\lambda^2 - \lambda_2^2) + G_3 \lambda^2 / (\lambda^2 - \lambda_3^2)$$

in which G_1, G_2, G_3 and $\lambda_1, \lambda_2,$ and λ_3 are constants (called Sellmeier coefficients) that are determined by fitting this expression to the experimental data. The table below gives the Sellmeier coefficients for pure silica (SiO_2) and SiO_2 -13.5 mol.% GeO_2 . Write a program on your computer or calculator, or a math software package or even a spread sheet program to obtain the refractive index n as a function of λ from 0.5 μm to 1.8 μm for both pure silica and SiO_2 -13.5 mol.% GeO_2 . Obtain the group index, N_g , vs. wavelength for both materials and plot it on the same graph. Find the wavelength at which the material dispersion becomes zero in each material.

Material	G_1	G_2	G_3	λ_1	λ_2	λ_3
SiO_2	0.696749	0.408218	0.890815	0.0690660	0.115662	9.900559
$\text{SiO}_2/\text{GeO}_2$	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478

1.5 **TIR and Brewster's angle at water-air interface.**

- (a) Given that the refractive index of water is about 1.33, what is Brewster's angle for light traveling in air and reflected from the surface of the water?

- (b) Consider a diver in a sea pointing a flashlight towards the surface of the water. What is the critical angle for the light beam to be reflected from the water surface?

1.6 V-number and the number of modes. Estimate the number of modes that can be supported in a planar dielectric waveguide that is $100 \mu\text{m}$ wide and has $n_1 = 1.490$ and $n_2 = 1.470$ at the free-space source wavelength (λ), which is $1 \mu\text{m}$. Compare your estimate with the formula

$$M = \text{Int}(2V/\pi) + 1$$

1.7 A single mode fiber. A typical single mode optical fiber has a core of diameter $8 \mu\text{m}$ and a refractive index of 1.46 . The normalized index difference is 0.3% . The cladding diameter is $125 \mu\text{m}$. Calculate the numerical aperture and the acceptance angle of the fiber. What is the single mode cut-off wavelength λ_c of the fiber?